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at a single point, is the most natural one. Would it not be more appropriate, in view of the suggestions of mathematical physics, to suppose that the history of the universe is contained in a vast system of differential equations, or in view of Volterra's hypothesis of "heredity," of integro-differential or other still more complicated equations, so that the knowledge of a *finite* set of initial conditions would suffice to determine a solution completely?

Also the notion of the "most probable" of all possible trends of natural phenomena may be much less simple than special cases would indicate. That determination of the history of a country or the world which will produce the "most probable" of all conceivable birth-rates may not coincide with the determination which will produce the "most probable" duration of life. This almost obvious comment is not meant as an objection to Professor Weaver's remarks, nor is it implied that a "most probable" configuration under the whole imaginable set of auxiliary conditions may not also exist; such questions would naturally have to be settled if the present vision should come within appreciable range of reality.

The second discussion brings us back to something more concrete. Dr. Sensenig indicates a form of derivation of the integral as the limit of a sum, which practically amounts to a proof, for the case of an analytic function, of the identity of the concepts *integral* and *anti-derivative*. It is at times useful to have such proofs at hand, even in elementary instruction, as aids in producing conviction in the minds of students. Of course, the use of the formula for Σn^m will prove an obstacle to the use of Dr. Sensenig's work for that purpose. It is doubtless universally familiar that the convergence of the Cauchy sum, in the case of a monotonically increasing or decreasing continuous function, can be cast in a geometric form so vivid as to be accepted with ease by the ordinary classes in calculus.

As the third discussion appears an alternative derivation of the expressions for the half-angles of a plane triangle in terms of the sides. Professor Baudin obtains the relations from the law of sines instead of from the law of cosines. The work is less simple than the ordinary methods; it is interesting, however, to see that it can be carried out.

I. FORECAST.

By WARREN WEAVER, California Institute of Technology.

The foundations of science and scientific thought have been subjected in the last fifty years to examination of a most critical sort. This examination has resulted in development along two general lines; a filling of needful matter into the interstices of the even yet porous body of logical thought where the general trend of the previously accepted body has been found still tenable, and an opening up of new problems in those regions where the old theories have been found inadequate or merely approximate. As an example of the first type there comes to one's mind the rigor that has been brought to the fundamental concepts of the calculus by means of function theory. The outstanding illustration of the latter type of problem of the newer science is the recasting of the old concepts of time and space that is necessary under the modern theory of relativity, along with the fundamental and far-reaching alterations which this theory imposes upon mechanics and electrical theory.

In many such investigations a mode of attack is being used that reminds one of the older metaphysical and theological considerations of scientific problems, such as the proof of the Principle of Least Action given by Maupertius. As the type of problem being studied becomes more and more fundamental and ultimate in nature one is not surprised to find that philosophical considerations enter along with considerations of a more formal mathematical nature. So that it is a little less idle than hitherto for one to speculate how fundamental the truths really may be that he meets with in pure mathematics and in the mathematics of pure physics. Some of the theories that have hitherto seemed restricted to the formal considerations of the mathematician have indeed been emerging of late as truths the generality and broad applicability of which are not yet fully appreciated. Out of an almost indecent past shadily connected with the gaming table comes a theory of probability that, expanded by the researches of Gibbs and others, furnishes the one ultimate foundation for the kinetic theory of gases, kinetic theories of electrical conductivity, a broad class of thermodynamical problems, and the theory of statistics in all its branches. The fact that of all possible complexions of a system the trend of nature is always towards the most probable seems to be the statement of a principle whose breadth has been only of late and perhaps not even now fully appreciated. One may easily cite other modern questions to which one can give a broadly philosophical cast. As has been suggested in a recent note in *Science*[1] velocity is essentially of such nature that there is a *best* way to measure it, a conclusion which, if completely acceptable, may be taken as a fundamental argument for the relativity theory. The work of Dr. R. C. Tolman in developing what he has called the Principle of Similitude[2] has furnished upon some such general ground not only a startlingly immediate method of establishing many physical relationships known upon other grounds to be correct, but seems also to disclose these results as *inevitable* truths in a way that a more formal proof does not at all. The atomic structure of matter and the corpuscular nature of energy seem to be demanded by the fact that if nature is to tend towards some certain definite most probable configuration and a certain definite most probable distribution of energy it must be possible to recognize discrete elements of matter and energy.[3] Generalized relativity has been extended along certain lines to suggest that it may be the expression of a very fundamental truth that possibly and probably transcends the sciences of mathematics and physics, narrowly conceived, and is applicable to diversified considerations regarding life.[1] Dr. H. Bateman has suggested, in the note in the *Philosophical Magazine* to which reference has just been made, the possible and probable existence of a deep underlying principle which, since it will be the ultimately true equation of motion, not only would furnish the results of ordinary relativity theory, but would also contain, among other things, an explanation of the fact that good designs are perpetuated in nature. We may conclude therefore that the trend of scientific thought at the present time renders it more hopeful and less inexcusable to contemplate analogies between the principles of mathematics and the principles of life, in the attempt to enrich and perhaps shed new light upon each.

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[1] "The Nature of Velocity" by Tenney L. Davis, *Science*, Oct. 10, 1919.

[2] Articles on the Principle of Similitude and related subjects: *Physical Review*, April, 1914; Aug., 1914; *Phys. Rev.*, 6, 219 (1915); July, 1916; *Phys. Rev.*, 9, 237 (1917).

[3] *Eight Lectures on Theoretical Physics* by Max Planck, pp. 42–55. Columbia University Press.

It ought to seem queer to the scientist that history is so nearly exclusively a record of what man has done, and in so very small measure a study of what man will do. For the method of the scientist is to record past events only so far as is necessary to the accumulation of sufficient data to check theories which make possible the prediction of events in the future. It has probably been many times suggested that serious study be made of the possibility of a similar method being used in history, but perhaps the best-known comes from the versatile pen of H. G. Wells. In a discourse entitled "The Discovery of the Future"[2] delivered at the Royal Institution in London he gives his foundation for an optimistic hope that a bold and searching study may serve to give us eventually a good deal of general but exceedingly useful knowledge of the future. He says that "as one assimilates the broad conception of science, the persuasion comes into one's mind that the adequacy of causation is universal," so that "the man of science comes to believe at last that the events of the year A.D. 4000 are as fixed, settled, and unchangeable as the events of the year A.D. 1600." He cites the pushing back of our information to prehistoric times by means not of revelation, but by means of a keen and vigorous habit of inquiry, and continues; "and now if it has been possible for men by picking out a number of suggestive and significant looking things in the present, by comparing them, criticizing them, and discussing them, with a perpetual insistence upon 'Why?', without any guiding tradition and indeed in the teeth of established beliefs, to construct this amazing searchlight of inference into the remoter past, is it really, after all, such an extravagant thing to suggest that, by seeking operating causes instead of fossils and by criticizing them as consistently and thoroughly as the geologic record has been criticized it may be possible to throw a searchlight of inference forward instead of backward, and to attain a knowledge of coming things as clear, as universally convincing, and infinitely more important to mankind than the clear vision of the past geology has opened to us during the nineteenth century?" The lecture includes a discussion to show that the existence of the "exceptional man," such as Napoleon, Cæsar, or William the Conqueror need not hopelessly remove the possibility for success of such a scheme, and it is careful not to hold out any fantastic hope for individual prophecy—a fact which need no more disturb the main result than the fact that we are unable to predict the motion of any single molecule in a gas need preclude the possibility of a statistical forecast of a future state of it. We can no more hope to forecast the life of a single certain man than, to extend Wells's illustration, we can hope to give a detailed history of the past life of some single certain labyrinthodon or pterodactyl.

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[1] See, for example, the end of an article called "The Physical Aspect of Time," by H. Bateman, *Memoirs and Proc. of Manchester Literary and Philosophical Soc.*, Vol. 54, Part 3, 1909–1910: and, same author, *Phil. Mag.*, Vol. 37, Feb., 1919.

[2] Published by B. W. Huebsch, New York, in book form with the above title.

A certain crude sort of mathematical justification for some such notion had occurred to the writer about a year before he chanced to see this book of Wells. An analytic function y of an independent variable x may be expanded by means of Taylor's Theorem in a power series in $(x - a)$. If this series represents the function y for values x to the left of a, it will also represent y for values of x to the right of a. In fact, the series will represent the function y for *all* values of x if it is convergent for all real values of x. And all we have to know in order to possess complete information about this series is the value of the function and of its derivatives *at the point* a. If we call the independent variable t instead of x the analogy becomes apparent. It clearly suggests that if we could know, about a certain social factor, its present state, and have complete instantaneous information as to the trend of it we ought to be able to forecast accurately its importance at any future time.

One may conjecture some general things about certain types of functions that would probably prove necessary in such a connection. A general steady growth would be represented by a linear function, to determine which we would need only its present value, and present rate of change. The sort of thing of which one says that "history repeats itself" would be represented by circular or other periodic functions. One may guess that several such, of different periods, would combine to produce nearly zero result for long stretches of time, and then with a sudden "beat," through constructive addition, give a sharp maximum—perhaps we may say a Renaissance! Any factor which is being damped out would in all probability be represented by an ordinary exponential damping term, with negative exponent. It is conceivable that little else would be necessary, and in all these cases it is cheerful to note that our information would extend to *any* time in the future, however remote; which corresponds directly to the mathematical fact that the Maclaurin expansions of the functions kt, $\sin t$, $\cos t$, and e^{-kt} are convergent for all real values of t. We might conceive of a Fourier series (perhaps multiplied by an exponential factor) doing the trick for us, so that some later-day genius could construct for us a tide-predicting machine not for the ocean's tides, but for the tides of human thought and the surge of human progress.

A possible objection to the application of the above analogy lies in the fact that one needs, in general, the value of an infinite number of derivatives at the point a really to possess the whole series. Beyond the fact that a finite number of terms (and often a small number) gives a good approximation for short ranges of x mathematics has a further hopeful suggestion. In the potential theory it is very well known that if we have given, over a closed surface S, the value at every point of the potential function V we can calculate the potential at any other point.[1] In terms of our former analogy we may say that if we have a set of "cross-section" values of a function (which in our application would be a set of values for a constant *t*) and of its first derivative, and know that the function is characterized and specialized by the fact that it satisfies some very general condition (which condition while very general must get at the very root of the matter) we can completely determine the function at all other points. This general condition, in the case of ordinary potential and many allied problems is, of course, given by the ordinary Laplacian equation $\Delta^2 V = 0$. This suggests that if, by careful study, we could but obtain some universally applicable and fundamentally far-reaching principle—some truly dynamic theory of history, to use Henry Adams's phrase, we could probably thus cut down enormously upon the amount of contemporary data necessary.

This analogy may perhaps all be interpreted as merely strengthening Wells's point that in science effect seems inevitably to follow cause in an unending and inescapable chain of events. There is perhaps no more striking example of the adequacy of causation than that contained in the fact that the value and behavior of y a million million miles down the x axis is inevitably determined by the way in which the curve crosses the y axis; and it is a natural and, it would seem, not ridiculous hope to expect that some day there will be an historian bold enough and wise enough to write down a few of the equations of history which will hold not only for the minus sign, but also for the plus.

II. A Proof of the Definite Integral Formula.

By Wayne Sensenig, Conshohocken, Pa.

On pages 214–216 of Lamb's *Infinitesimal Calculus* several special cases of the definite integral are calculated "ab initio." [2] The following direct proof of the formula for the definite integral covers cases where y can be expressed as a *convergent* power series in x over the *closed* interval considered in the definite integral. The area under a curve is used as an illustration, but the form of proof is obviously independent of any special geometric interpretation. Use is made of the following well-known formula:[3]

$$
\begin{aligned}
S_m &= 1^m + 2^m + \cdots + n^m \\
&= \frac{n^{m+1}}{m+1} + \tfrac{1}{2}n^m + B_1 \frac{m}{2!} n^{m-1} - B_3 \frac{m(m-1)(m-2)}{4!} n^{m-3} \\
&\quad + B_5 \frac{m(m-1)(m-2)(m-3)(m-4)}{6!} n^{m-5} - \cdots,
\end{aligned}
$$

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[1] If it is desired to calculate at points exterior to S the regularity of V at infinity is also necessary. The above mathematical fact does not correspond perfectly to the previous example, but the aspect of the logical content with which we are concerned is closely enough connected with the former to serve our purpose.

[2] Similar exercises are found in various text-books. Problems of the same character have from time to time appeared in the Monthly.—Editor.

[3] Hall and Knight, *Higher Algebra*, p. 337.